THE UNIVERSITY OF CHICAGO SCHOOL MATHEMATICS PROJECT

UCSMP

Texts for Grades 6 to 12

2021-22 Catalog

UChicagoSolutions

THE NEW SOURCE FOR UCSMP TEXTS FOR GRADES 6 TO 12

uchicagosolutions.com

Table of Contents

At UChicagoSolutions we are continuing to refresh the current UCSMP courses for Grades 6 to 12. Our long-term goal is to rebuild the robust market presence needed to support a new edition of the entire curriculum. Our strategy is to continue adhering to the principles that have made UCSMP a twenty-five-year success story while addressing the realities of the Common Core State Standards.

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UCSMP

About UCSMP

About Us

The University of Chicago School Mathematics Project (UCSMP) was founded in 1983 with the goal of improving mathematics instruction in grades pre-K through 12. Affiliated with the University's Center for Elementary Mathematics and Science Education (CEMSE), UCSMP has been at the forefront of research- and testing-based curriculum development, evaluation, and implementation in mathematics for more than thirty years.

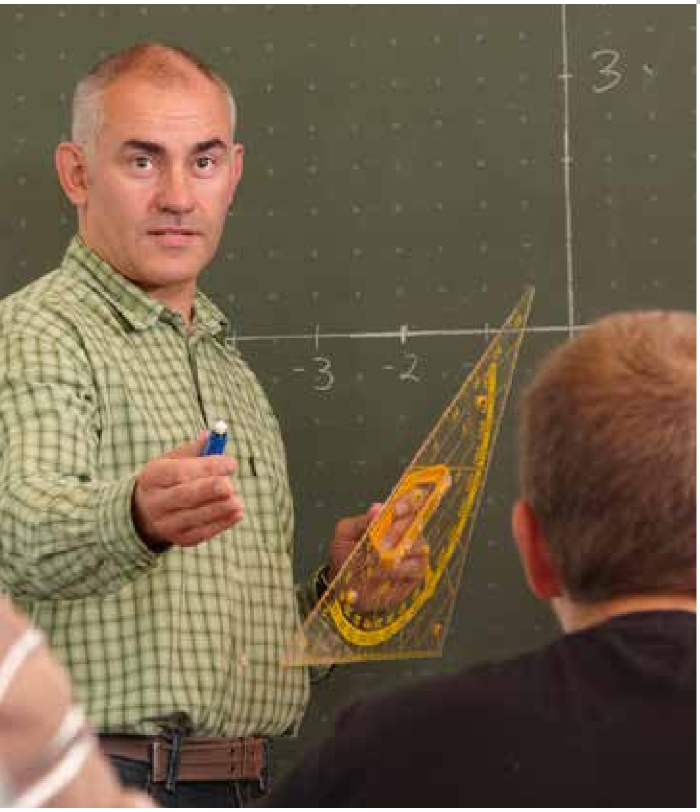
The Grades 6 to 12 Courses

UCSMP entered the market for middle and high school mathematics textbooks in 1989 with the first edition of the series. The second edition appeared in the mid-1990s, with the third-edition books rolling out between 2008 and 2010. Refreshed versions of the third edition published by UChicagoSolutions were initially introduced in 2015.

History of Success

Counting every course and edition, more than four million Student Editions of UCSMP for Grades 6 to 12 have been sold. UChicagoSolutions is proud to continue this legacy.

Instructional Approach



Unique Approach to Understanding

UCSMP for Grades 6 to 12 incorporates opportunities for students to develop mathematical skills and master concepts they will use every day. Concepts are introduced and reinforced using real-life applications. Our unique SPUR approach provides students with four dimensions of understanding, so they are able to develop methods for solving problems in different ways.

The SPUR approach is structured into every lesson as well as the Chapter Review questions and the Chapter Self-Test that appear at the end of each chapter.

Multi-Dimensional SPUR Approach to Understanding

Skills	Skills and algorithms with and without calculator and computer technology
Properties	Deductions, justifications, arguments, and proofs
Uses	Real-world applications and modeling
Representations	Representations of mathematics through symbols, pictures, diagrams, and graphs

Structured Lesson Review

Each lesson ends with a set of questions framed to allow students to develop their skills and extend their understanding. These concepts and questions are presented in a sequence we call CARE, which our research and testing has shown reinforces students' learning process.

Question Sets Created with CARE

Covering the Ideas	Basic understanding in the four dimensions of **SPUR**
Applying the Mathematics	Introduction of different contexts and connections with other mathematics
Review	For sustaining and increasing performance as well as previewing ideas needed in lessons to follow
Exploration	Going beyond the content of the lesson with a related idea

Implementing UCSMP

Key Features of the Program

FIELD TESTED: Every course has been field tested prior to publication.

CONTINUITY: UCSMP secondary courses are aligned with K–5 *Everyday Mathematics®*.

TECHNOLOGY: Calculator and computer usage is integrated into every course.

CONTEXT: Settings for activities and problems feature contemporary content and context.

ACTIVE LEARNING: Lessons have a focus on active learning, both individually and in groups.

READING INTEGRATION: Active reading that engages students and helps create life-long learners.

Alignment with *Everyday Mathematics®*

UCSMP provides an uninterrupted curriculum from pre-Kindergarten through grade 12. Because of close collaboration among UCSMP authors and editors, there is a smooth development and tight alignment of content across the grades. In particular, there is strong content articulation going from *Everyday Mathematics®* (published by McGraw-Hill) to *UCSMP Grades 6–12* (published by UChicagoSolutions).

Progression

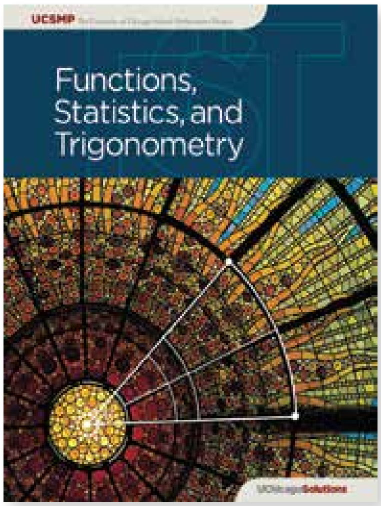
UCSMP courses are aligned to build on each other to form a seamless curriculum as illustrated in the table below.

Students can enter the UCSMP middle and high school curriculum at any point beginning at grade 5 depending on capability. They are advantaged at entry by previous exposure to UCSMP's primary grades mathematics curriculum, the *Everyday Mathematics®* series published by McGraw-Hill.

Each UCSMP course builds on its precursors, but the program is flexible enough to allow each course to be taught independently.

Grade	Top 10%–20% of Students	Next 50% of Students	Next 20% of Students	Remainder of Students
5	Pre-Transition Mathematics			
6	Transition Mathematics	Pre-Transition Mathematics		
7	Algebra	Transition Mathematics	Pre-Transition Mathematics	
8	Geometry	Algebra	Transition Mathematics	Pre-Transition Mathematics
9	Advanced Algebra	Geometry	Algebra	Transition Mathematics
10	Functions, Statistics & Trigonometry	Advanced Algebra	Geometry	Algebra
11	Precalculus & Discrete Mathematics	Functions, Statistics & Trigonometry	Advanced Algebra	Geometry
12	Calculus	Precalculus & Discrete Mathematics	Functions, Statistics & Trigonometry	Advanced Algebra

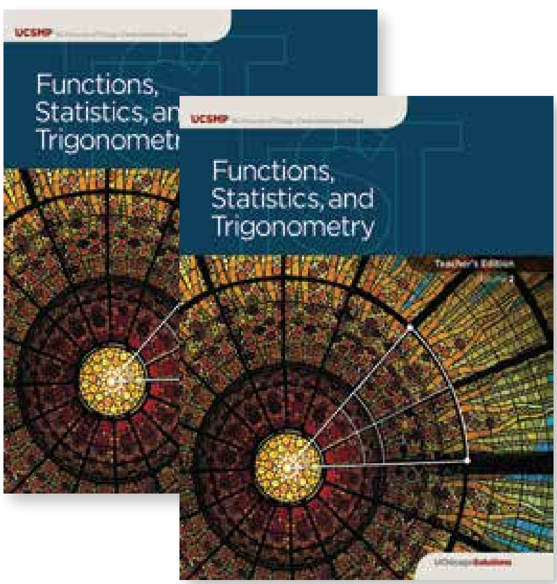
UCSMP Program Components



Each course in *UCSMP Grades 6–12* includes the following components (except where specified):

Student Edition

- Broad-based, reality-oriented, and easy-to-understand approach to mathematics
- Emphasis on reading so students learn how to learn mathematics
- Powerful technology integrated throughout
- New online edition

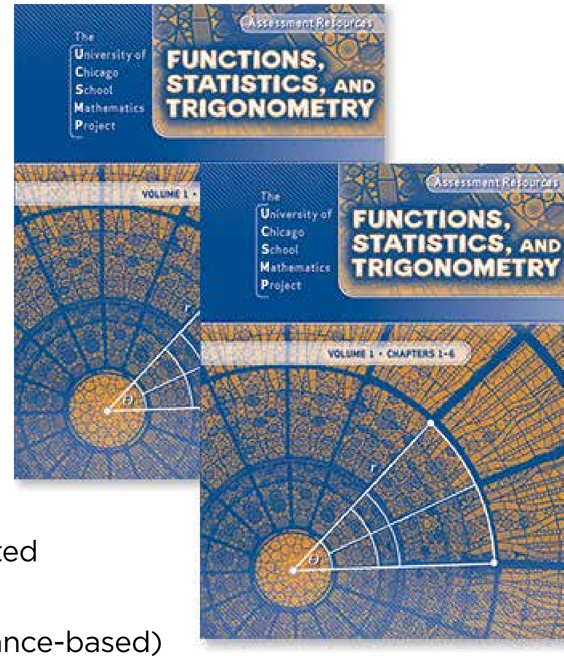


Teacher's Edition

- Mathematical Background for each lesson
- Four-step lesson plan
  - Warm-Up
  - Teaching
  - Assignment
  - Wrap-Up
- Differentiation Options

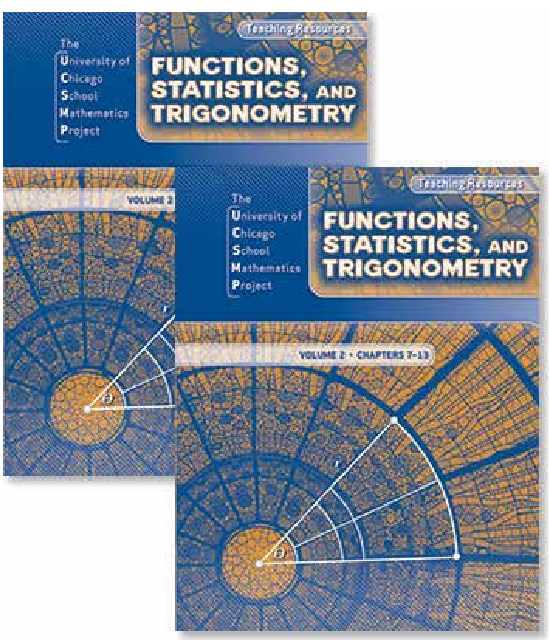
Implementation Guide

Teachers and schools new to UCSMP may find our Implementation Guide particularly useful. The 56-page booklet is filled with insights and tips on effective instructional approaches and assistance in developing an effective pedagogy.



Assessment Resources

- Soft-cover two-volume set with perforated pages for easy reproduction
- Quizzes (two per chapter)
- Chapter Tests (five forms)
  – Forms A and B (constructed response; parallel forms)
  – Forms C and D (performance-based)
  – Cumulative Form
- Comprehensive Tests (four per course; multiple-choice format)
- Answers or Evaluation Guides for all quizzes and tests; correlation of SPUR Objectives to Chapter Tests Forms A–D
- Assessment forms (student- and teacher-completed)



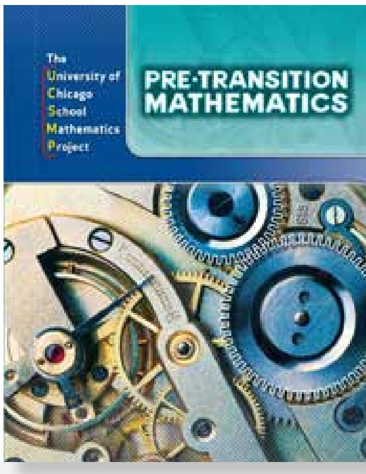
Teaching Resources

- Lesson Masters (one- and two-page practice and review blackline masters; overprinted answers included)
- Resource Masters (generic teaching aids, copies of all Teacher's Edition Warm-Ups and Additional Examples, and more)
- Game Masters (for *Pre-Transition Mathematics* and *Transition Mathematics* only)

GRADES 5 – 8

Pre-Transition Mathematics

REAL-WORLD PROBLEMS SHOW PATTERNS AND RELATIONSHIPS

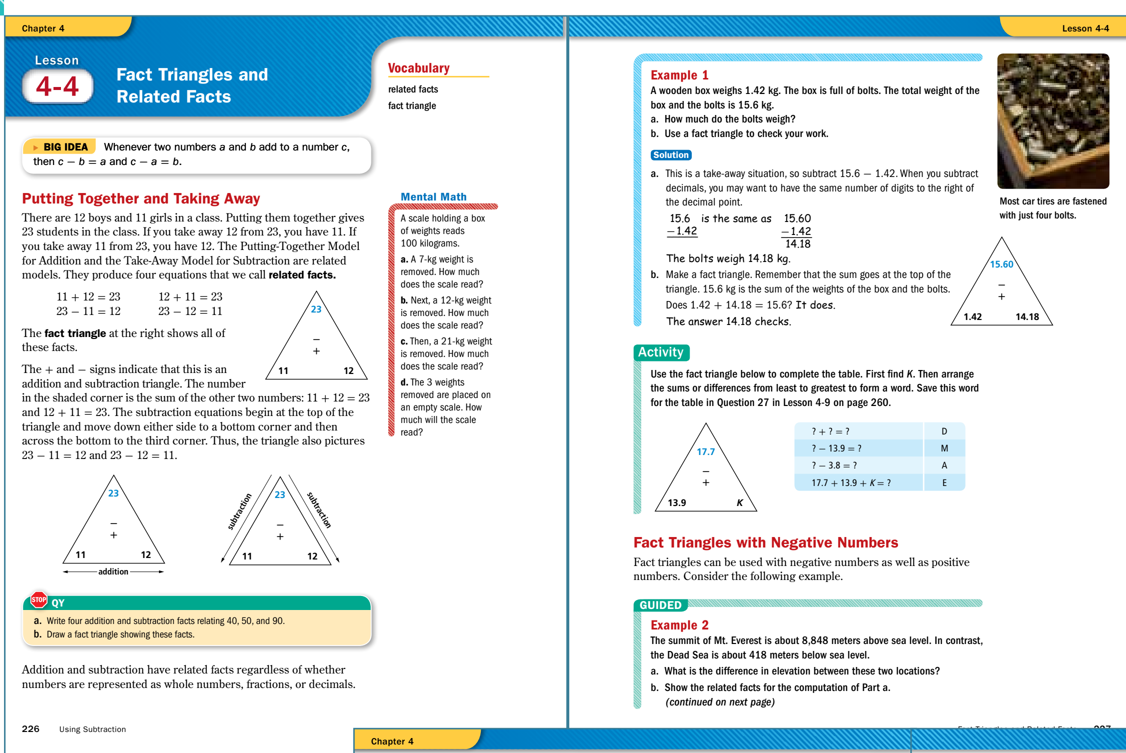


The goals of *Pre-Transition Mathematics* are to take an in-depth approach to the arithmetic of rational numbers and to extend the basic ideas of algebra, geometry, probability, and statistics for students from *Everyday Mathematics*® or to introduce these basic ideas for students from non-UCSMP programs who may not have been previously exposed to them.

Pre-Transition Mathematics is intended primarily for students who are ready for a 6th-grade curriculum. It reflects the practice of identifying and working on uses of numbers and operations that characterized the earlier editions of *Transition Mathematics*. Fractions and percents are particularly emphasized. There is also a major emphasis on dealing with data and geometry. Algebra is integrated throughout the text as a way of describing generalizations, as a language for formulas, and as an aid in solving simple equations. The text is characterized by rich problems throughout.

Table of Contents

UCSMP Grades 6–12 is a flexible program, allowing schools to offer the appropriate mathematics to students regardless of their grade level.



Chapter 4

Lesson 4-4

Lesson 4-4

Fact Triangles and Related Facts

Vocabulary

related facts

fact triangle

BIG IDEA Whenever two numbers a and b add to a number c, then $c - b = a$ and $c - a = b$.

Putting Together and Taking Away

There are 12 boys and 11 girls in a class. Putting them together gives 23 students in the class. If you take away 12 from 23, you have 11. If you take away 11 from 23, you have 12. The Putting-Together Model for Addition and the Take-Away Model for Subtraction are related models. They produce four equations that we call **related facts.**

$11 + 12 = 23$ $12 + 11 = 23$

$23 - 11 = 12$ $23 - 12 = 11$

The **fact triangle** at the right shows all of these facts.

The + and − signs indicate that this is an addition and subtraction triangle. The number in the shaded corner is the sum of the other two numbers: $11 + 12 = 23$ and $12 + 11 = 23$. The subtraction equations begin at the top of the triangle and move down either side to a bottom corner and then across the bottom to the third corner. Thus, the triangle also pictures $23 - 11 = 12$ and $23 - 12 = 11$.

Mental Math

A scale holding a box of weights reads 100 kilograms.

a. A 7-kg weight is removed. How much does the scale read?

b. Next, a 12-kg weight is removed. How much does the scale read?

c. Then, a 21-kg weight is removed. How much does the scale read?

d. The 3 weights removed are placed on an empty scale. How much will the scale read?

QY

a. Write four addition and subtraction facts relating 40, 50, and 90.

b. Draw a fact triangle showing these facts.

Addition and subtraction have related facts regardless of whether numbers are represented as whole numbers, fractions, or decimals.

226 Using Subtraction

Example 1

A wooden box weighs 1.42 kg. The box is full of bolts. The total weight of the box and the bolts is 15.6 kg.

a. How much do the bolts weigh?

b. Use a fact triangle to check your work.

Solution

a. This is a take-away situation, so subtract 15.6 − 1.42. When you subtract decimals, you may want to have the same number of digits to the right of the decimal point.

15.6 is the same as 15.60; $15.60 - 1.42 = 14.18$

The bolts weigh 14.18 kg.

b. Make a fact triangle. Remember that the sum goes at the top of the triangle. 15.6 is the sum of the weights of the box and the bolts.

Does 1.42 + 14.18 = 15.6? It does.

The answer 14.18 checks.

Most car tires are fastened with just four bolts.

Activity

Use the fact triangle below to complete the table. First find K. Then arrange the sums or differences from least to greatest to form a word. Save this word for the table in Question 27 in Lesson 4-9 on page 260.

? + ? = ?	D
? − 13.9 = ?	M
? − 3.8 = ?	A
17.7 + 13.9 + K = ?	E

Fact Triangles with Negative Numbers

Fact triangles can be used with negative numbers as well as positive numbers. Consider the following example.

GUIDED

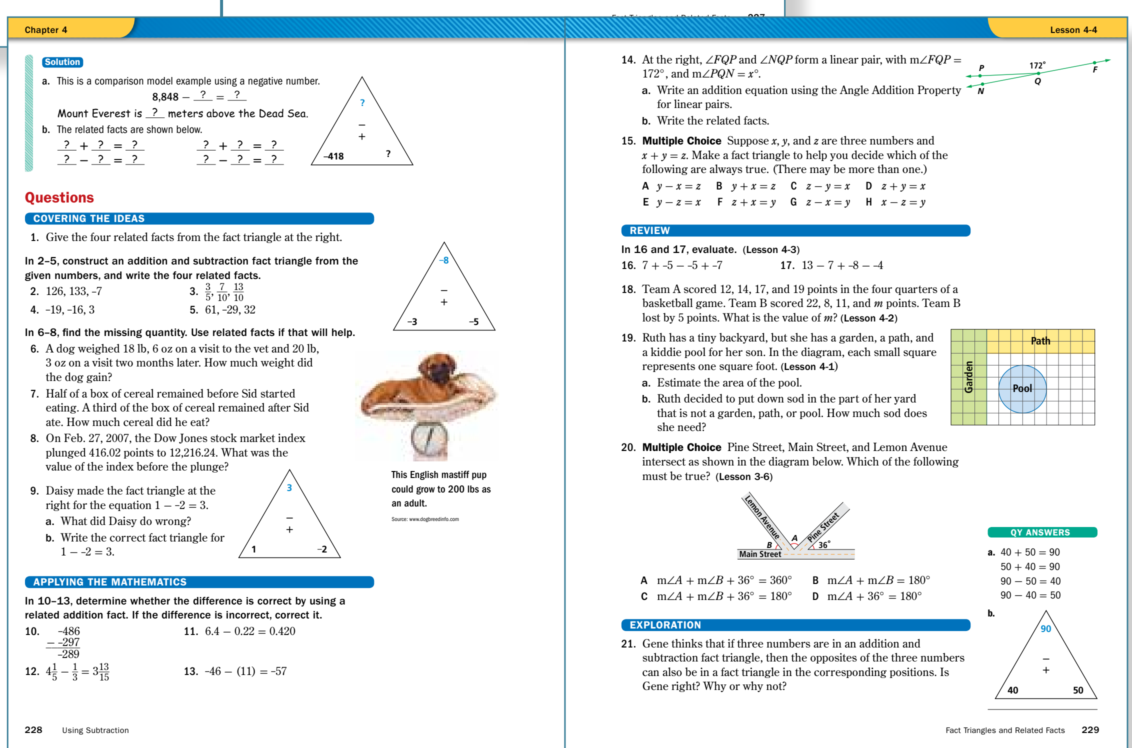
Example 2

The summit of Mt. Everest is about 8,848 meters above sea level. In contrast, the Dead Sea is about 418 meters below sea level.

a. What is the difference in elevation between these two locations?

b. Show the related facts for the computation of Part a.

(continued on next page)



Chapter 4

Lesson 4-4

Solution

a. This is a comparison model example using a negative number.

8,848 − _?_ = _?_

Mount Everest is _?_ meters above the Dead Sea.

b. The related facts are shown below.

_?_ + _?_ = _?_ _?_ + _?_ = _?_

_?_ − _?_ = _?_ _?_ − _?_ = _?_

Questions

COVERING THE IDEAS

1. Give the four related facts from the fact triangle at the right.

In 2–5, construct an addition and subtraction fact triangle from the given numbers, and write the four related facts.

2. 126, 133, -7
3. $\frac{3}{5}, \frac{7}{10}, \frac{13}{10}$
4. −19, −16, 3
5. 61, −29, 32

In 6–8, find the missing quantity. Use related facts if that will help.

6. A dog weighed 18 lb, 6 oz on a visit to the vet and 20 lb, 3 oz on a visit two months later. How much weight did the dog gain?
7. Half of a box of cereal remained before Sid started eating. A third of the box of cereal remained after Sid ate. How much cereal did he eat?
8. On Feb. 27, 2007, the Dow Jones stock market index plunged 416.02 points to 12,216.24. What was the value of the index before the plunge?

This English mastiff pup could grow to 200 lbs as an adult.

Source: www.dogbreedinfo.com

9. Daisy made the fact triangle at the right for the equation $1 - -2 = 3$.
   a. What did Daisy do wrong?
   b. Write the correct fact triangle for $1 - -2 = 3$.

APPLYING THE MATHEMATICS

In 10–13, determine whether the difference is correct by using a related addition fact. If the difference is incorrect, correct it.

10. $-486 - -297 = -289$
11. $6.4 - 0.22 = 0.420$
12. $4\frac{1}{5} - \frac{1}{3} = 3\frac{13}{15}$
13. $-46 - (11) = -57$

228 Using Subtraction

14. At the right, $\angle FQP$ and $\angle NQP$ form a linear pair, with $m\angle FQP = 172°$, and $m\angle PQN = x°$.
   a. Write an addition equation using the Angle Addition Property for linear pairs.
   b. Write the related facts.

15. **Multiple Choice** Suppose x, y, and z are three numbers and $x + y = z$. Make a fact triangle to help you decide which of the following are always true. (There may be more than one.)

A $y - x = z$ B $y + x = z$ C $z - y = x$ D $z + y = x$

E $y - z = x$ F $z + x = y$ G $z - x = y$ H $x - z = y$

REVIEW

In 16 and 17, evaluate. (Lesson 4-3)

16. $7 + -5 - -5 + -7$
17. $13 - 7 + -8 - -4$

18. Team A scored 12, 14, 17, and 19 points in the four quarters of a basketball game. Team B scored 22, 8, 11, and m points. Team B lost by 5 points. What is the value of m? (Lesson 4-2)

19. Ruth has a tiny backyard, but she has a garden, a path, and a kiddie pool for her son. In the diagram, each small square represents one square foot. (Lesson 4-1)
   a. Estimate the area of the pool.
   b. Ruth decided to put down sod in the part of her yard that is not a garden, path, or pool. How much sod does she need?

20. **Multiple Choice** Pine Street, Main Street, and Lemon Avenue intersect as shown in the diagram below. Which of the following must be true? (Lesson 3-6)

A $m\angle A + m\angle B + 36° = 360°$ B $m\angle A + m\angle B = 180°$

C $m\angle A + m\angle B + 36° = 180°$ D $m\angle A + 36° = 180°$

EXPLORATION

21. Gene thinks that if three numbers are in an addition and subtraction fact triangle, then the opposites of the three numbers can also be in a fact triangle in the corresponding positions. Is Gene right? Why or why not?

QY ANSWERS

a. $40 + 50 = 90$
$50 + 40 = 90$
$90 - 50 = 40$
$90 - 40 = 50$

b.

Fact Triangles and Related Facts 229

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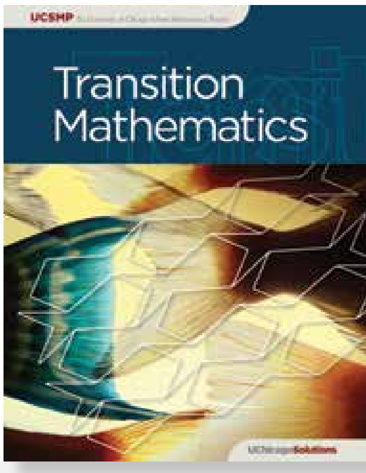
GRADES 6 – 9

Transition Mathematics

STUDENTS WORK WITH SPREADSHEETS AND DYNAMIC GEOMETRY SYSTEMS



Transition Mathematics acts as a stepping-stone from the processes learned in _Pre-Transition Mathematics_ or Sixth Grade _Everyday Mathematics_ to the material presented in UCSMP _Algebra_ and UCSMP _Geometry. Transition Mathematics_ incorporates applied arithmetic, algebra, and geometry and connects these to measurement, probability, and statistics.

Variables are used to generalize patterns, as abbreviations in formulas, and as unknowns in problems, and are represented on the number line and graphed in the coordinate plane. Basic arithmetic and algebraic skills are connected to corresponding geometry topics. This course provides opportunities for students to visualize and demonstrate concepts with a focus on real-world applications. Graphing calculators are assumed for home use.

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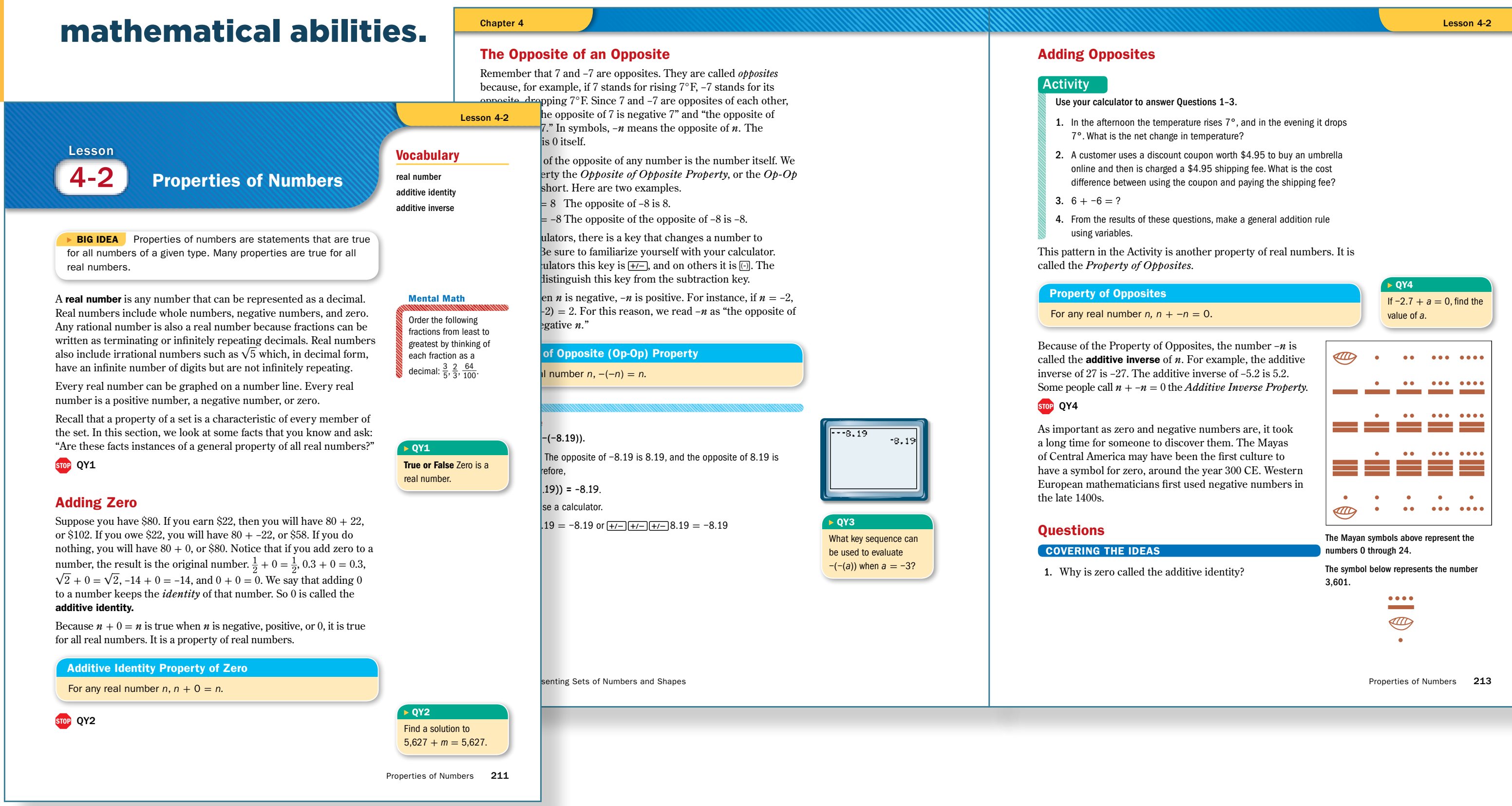
Games enable students to practice important mathematical skills while gaining confidence in their mathematical abilities.



Lesson 4-2

Lesson

4-2 Properties of Numbers

Vocabulary

real number
additive identity
additive inverse

BIG IDEA Properties of numbers are statements that are true for all numbers of a given type. Many properties are true for all real numbers.

A **real number** is any number that can be represented as a decimal. Real numbers include whole numbers, negative numbers, and zero. Any rational number is also a real number because fractions can be written as terminating or infinitely repeating decimals. Real numbers also include irrational numbers such as $\sqrt{5}$ which, in decimal form, have an infinite number of digits but are not infinitely repeating.

Every real number can be graphed on a number line. Every real number is a positive number, a negative number, or zero.

Recall that a property of a set is a characteristic of every member of the set. In this section, we look at some facts that you know and ask: "Are these facts instances of a general property of all real numbers?"

STOP QY1

Mental Math

Order the following fractions from least to greatest by thinking of each fraction as a decimal: $\frac{3}{5}, \frac{2}{3}, \frac{64}{100}$.

QY1

True or False Zero is a real number.

Adding Zero

Suppose you have \$80. If you earn \$22, then you will have 80 + 22, or \$102. If you owe \$22, you will have 80 + -22, or \$58. If you do nothing, you will have 80 + 0, or \$80. Notice that if you add zero to a number, the result is the original number. $\frac{1}{2} + 0 = \frac{1}{2}$, $0.3 + 0 = 0.3$, $\sqrt{2} + 0 = \sqrt{2}$, $-14 + 0 = -14$, and $0 + 0 = 0$. We say that adding 0 to a number keeps the *identity* of that number. So 0 is called the **additive identity.**

Because $n + 0 = n$ is true when n is negative, positive, or 0, it is true for all real numbers. It is a property of real numbers.

Additive Identity Property of Zero

For any real number n, $n + 0 = n$.

STOP QY2

QY2

Find a solution to $5{,}627 + m = 5{,}627$.

Properties of Numbers 211

Chapter 4

The Opposite of an Opposite

Remember that 7 and –7 are opposites. They are called *opposites* because, for example, if 7 stands for rising 7°F, –7 stands for its opposite, dropping 7°F. Since 7 and –7 are opposites of each other, ...he opposite of 7 is negative 7" and "the opposite of ...7." In symbols, $-n$ means the opposite of n. The ...is 0 itself.

...of the opposite of any number is the number itself. We ...erty the *Opposite of Opposite Property*, or the *Op-Op* ...short. Here are two examples.

...= 8 The opposite of –8 is 8.

...= –8 The opposite of the opposite of –8 is –8.

...ulators, there is a key that changes a number to ...Be sure to familiarize yourself with your calculator. ...ulators this key is [+/–], and on others it is [(–)]. The ...distinguish this key from the subtraction key.

...en n is negative, $-n$ is positive. For instance, if $n = -2$, ...-2) = 2. For this reason, we read $-n$ as "the opposite of ...egative n."

...of Opposite (Op-Op) Property

...l number n, $-(-n) = n$.

...–(–8.19)).

...The opposite of –8.19 is 8.19, and the opposite of 8.19 is ...refore,

....19)) = –8.19.

...se a calculator.

....19 = –8.19 or [+/–][+/–][+/–]8.19 = –8.19

QY3

What key sequence can be used to evaluate $-(-(a))$ when $a = -3$?

...senting Sets of Numbers and Shapes

Lesson 4-2

Adding Opposites

Activity

Use your calculator to answer Questions 1–3.

1. In the afternoon the temperature rises 7°, and in the evening it drops 7°. What is the net change in temperature?
2. A customer uses a discount coupon worth \$4.95 to buy an umbrella online and then is charged a \$4.95 shipping fee. What is the cost difference between using the coupon and paying the shipping fee?
3. $6 + -6 = ?$
4. From the results of these questions, make a general addition rule using variables.

This pattern in the Activity is another property of real numbers. It is called the *Property of Opposites.*

Property of Opposites

For any real number n, $n + -n = 0$.

QY4

If $-2.7 + a = 0$, find the value of a.

Because of the Property of Opposites, the number $-n$ is called the **additive inverse** of n. For example, the additive inverse of 27 is –27. The additive inverse of –5.2 is 5.2. Some people call $n + -n = 0$ the *Additive Inverse Property.*

STOP QY4

As important as zero and negative numbers are, it took a long time for someone to discover them. The Mayas of Central America may have been the first culture to have a symbol for zero, around the year 300 CE. Western European mathematicians first used negative numbers in the late 1400s.

The Mayan symbols above represent the numbers 0 through 24.

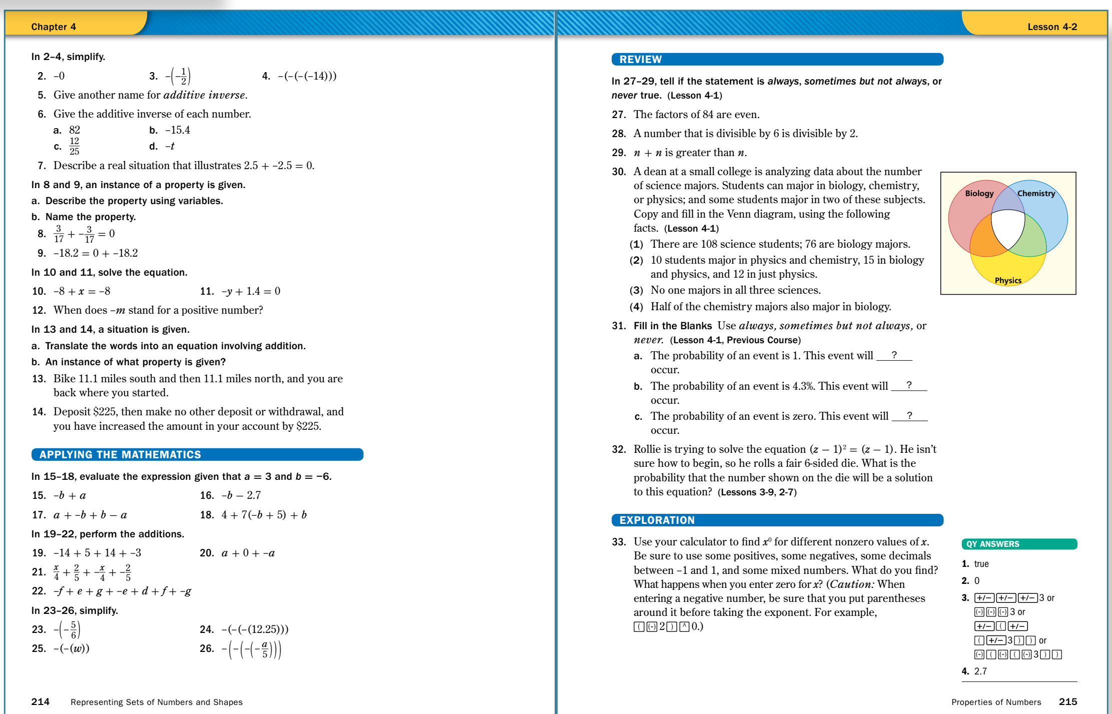
The symbol below represents the number 3,601.

Questions

COVERING THE IDEAS

1. Why is zero called the additive identity?

Properties of Numbers 213



Chapter 4

In 2–4, simplify.

2. –0
3. $-\left(-\frac{1}{2}\right)$
4. –(–(–(–14)))
5. Give another name for *additive inverse.*
6. Give the additive inverse of each number.
   a. 82
   b. –15.4
   c. $\frac{12}{25}$
   d. $-t$
7. Describe a real situation that illustrates $2.5 + -2.5 = 0$.

In 8 and 9, an instance of a property is given.
a. Describe the property using variables.
b. Name the property.

8. $\frac{3}{17} + -\frac{3}{17} = 0$
9. $-18.2 = 0 + -18.2$

In 10 and 11, solve the equation.

10. $-8 + x = -8$
11. $-y + 1.4 = 0$
12. When does $-m$ stand for a positive number?

In 13 and 14, a situation is given.
a. Translate the words into an equation involving addition.
b. An instance of what property is given?

13. Bike 11.1 miles south and then 11.1 miles north, and you are back where you started.
14. Deposit \$225, then make no other deposit or withdrawal, and you have increased the amount in your account by \$225.

APPLYING THE MATHEMATICS

In 15–18, evaluate the expression given that $a = 3$ and $b = -6$.

15. $-b + a$
16. $-b - 2.7$
17. $a + -b + b - a$
18. $4 + 7(-b + 5) + b$

In 19–22, perform the additions.

19. $-14 + 5 + 14 + -3$
20. $a + 0 + -a$
21. $\frac{x}{4} + \frac{2}{5} + -\frac{x}{4} + -\frac{2}{5}$
22. $-f + e + g + -e + d + f + -g$

In 23–26, simplify.

23. $-\left(-\frac{5}{6}\right)$
24. –(–(–(12.25)))
25. $-(-(w))$
26. $-\left(-\left(-\left(-\frac{a}{5}\right)\right)\right)$

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Lesson 4-2

REVIEW

In 27–29, tell if the statement is *always, sometimes but not always,* or *never* true. (Lesson 4-1)

27. The factors of 84 are even.
28. A number that is divisible by 6 is divisible by 2.
29. $n + n$ is greater than n.
30. A dean at a small college is analyzing data about the number of science majors. Students can major in biology, chemistry, or physics; and some students major in two of these subjects. Copy and fill in the Venn diagram, using the following facts. (Lesson 4-1)
    (1) There are 108 science students; 76 are biology majors.
    (2) 10 students major in physics and chemistry, 15 in biology and physics, and 12 in just physics.
    (3) No one majors in all three sciences.
    (4) Half of the chemistry majors also major in biology.

31. **Fill in the Blanks** Use *always, sometimes but not always,* or *never.* (Lesson 4-1, Previous Course)
    a. The probability of an event is 1. This event will __?__ occur.
    b. The probability of an event is 4.3%. This event will __?__ occur.
    c. The probability of an event is zero. This event will __?__ occur.
32. Rollie is trying to solve the equation $(z - 1)^2 = (z - 1)$. He isn't sure how to begin, so he rolls a fair 6-sided die. What is the probability that the number shown on the die will be a solution to this equation? (Lessons 3-9, 2-7)

EXPLORATION

33. Use your calculator to find x^0 for different nonzero values of x. Be sure to use some positives, some negatives, some decimals between –1 and 1, and some mixed numbers. What do you find? What happens when you enter zero for x? (*Caution:* When entering a negative number, be sure that you put parentheses around it before taking the exponent. For example, [(][(–)]2[)][^]0.)

QY ANSWERS

1. true
2. 0
3. [+/–][+/–][+/–]3 or [(–)][(–)][(–)]3 or [+/–][(][+/–] [(][+/–]3[)][)] or [(–)][(][(–)][(][(–)]3[)][)]
4. 2.7

Properties of Numbers 215

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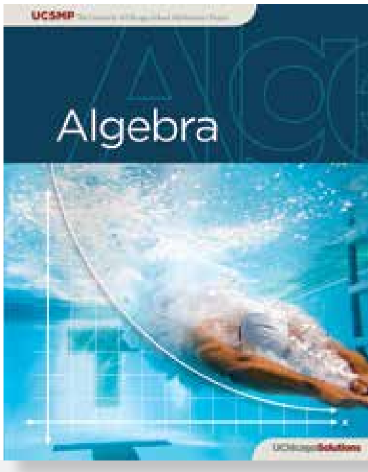
GRADES 7 – 10

Algebra

EMPHASIZES MULTIPLE APPROACHES TO EXPRESSIONS, EQUATIONS, AND FUNCTIONS



UCSMP *Algebra* introduces the language of algebra and the ways it is used in the real world, while integrating geometry, probability, and statistics with a variety of approaches and uses of contemporary technology.

UCSMP *Algebra* has a scope far wider than most other algebra texts, with mathematical topics integrated throughout. Statistics and geometry are settings for work with linear expressions and sentences. Probability provides a context for algebraic fractions and functions. Expressions, equations, and functions are described graphically, symbolically, and in tables. Concepts and skills are taught with a variety of approaches. Graphing calculators are assumed for home use, while computer algebra system (CAS) technology is used in the classroom to develop patterns and practice skills.

Table of Contents

By spreading out the traditional four-year high school curriculum over five years, *UCSMP Grades 6–12* gives students more time to learn the content they need.

Lesson

3-2 Solving Equations with Tables and Graphs

Vocabulary

solution to an equation

BIG IDEA Tables and graphs lead to exact or approximate solutions to equations.

Consider an equation that contains one variable, such as $2 + x = 6$. A **solution to an equation** is a value of the variable that makes the equation true. For $2 + x = 6$, 4 is a solution because $2 + 4 = 6$.

STOP QY1

Mental Math

Evaluate.

a. 50% of 120*P*

b. 5% of 120*P*

c. 50% of 12*P*

d. 5% of 12*P*

Solving with a Table and a Graph

In this lesson, you will learn two ways to solve equations: using a table and using a graph. We apply each way to solve $11 = 4x + 3$.

Example 1

Solve $11 = 4x + 3$ using a table.

Solution Replace *y* with 11 and make a table of values for $y = 4x + 3$. Look in the table to the right to see what value of *x* gives a value of 11 for *y*.

x	4 + 3	*y*
1	4(1) + 3	7
2	4(2) + 3	11
3	4(3) + 3	15
4	4(4) + 3	19
5	4(5) + 3	23

Looking at the table, you can see that when $x = 2$, $y = 11$. Based on the table, you can conclude that the solution to the equation $11 = 4x + 3$ is $x = 2$.

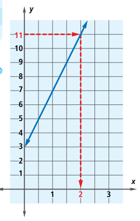
QY1

Is 5 a solution to $|-12m + 19| = 41$?

Example 2

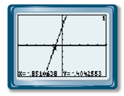
Solve $11 = 4x + 3$ using a graph.

Solution As in Example 1, replace *y* with 11. Then graph $y = 4x + 3$ and look for the value of *x* that corresponds with the value of 11 for *y*. To do this, start at 11 on the *y*-axis. Move right to the line, and then move down to the *x*-axis. As in Example 1, $x = 2$ is the solution.



(continued on next page)

You can also see this on a graphing calculator. Most graphing calculators have a trace option. The trace option allows you to move a cursor along the graph. As the cursor moves, it lists the coordinates on the graph. The calculator screen at the right shows the cursor at (–0.8510638, –0.4042553). Move the cursor until the *y*-coordinate is as close as possible to 11. Remember that the *x* value is only an approximation for the solution if the *y* value is also an approximation.



Situations Leading to Equations

Real-world situations can often be translated into relationships among numbers. If you can represent these relationships with an equation, then the methods of this and the next lesson can lead to a solution.

Example 3

Khalid lives in Alaska where there is no sales tax. He bought 3 tacos with a $5 bill and received $0.77 in change. Define the variable and represent the relationship with an equation.

Solution Let *t* represent the unknown cost of one taco. Because $5 minus the cost of 3 tacos equals $0.77 in change, $5 - 3t = 0.77$ is the equation.

STOP QY2

In 2006, the states without a sales tax were Alaska, Delaware, Montana, New Hampshire, and Oregon.

Source: Federation of Tax Administrators

QY2

Todd has $250 in his savings account. If he has a job that pays $11 per hour, how many working hours will it take for him to have enough money to buy a $481 surfboard?

Questions

COVERING THE IDEAS

1. Determine if the value of *x* is a solution for the equation.
   a. $11 - 9x = 47, x = 4$
   b. $3x^2 + 4x = 55, x = -5$
   c. $-x^3 = 64, x = -4$

In 2 and 3, solve by making a table for each equation and circling the row on the table that contains the solution.

2. $6 = 2x + 2$
3. $-5a + 7 = -23$

In 4 and 5, solve by making a graph for each equation and drawing lines from the *y*-axis to the graph, and then to the *x*-axis to indicate the solution.

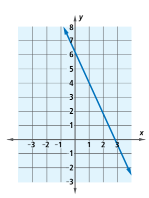
4. $y = -3x - 4, y = 2$
5. $y = 5x - 18, y = -3$
6. a. With a graphing calculator, make a table for $y = 5(5x - 2)$ using the integers from –5 to 0.
   b. Adjust the table to solve $5(5x - 2) = -26$.

In 7–9, define the variable and represent the relationship with an equation.

7. Solana lives where there is no sales tax. She bought five pieces of pizza for her friends with a $20 bill. She received $7.65 in change. What was the price of one piece of pizza?
8. Trevor is riding his bicycle across the United States. He started in a town on the East Coast and has already biked 630 miles. If he can ride 82 miles per day, how many days will it take him to complete the 3,210-mile journey?
9. The Coles are saving to send their child to college. They currently have $5,275 in the bank and are saving $950 per year. How many years will pass before they save $20,000?

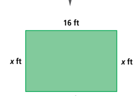
APPLYING THE MATHEMATICS

10. Use a graph to solve $5 - 3t = 0.77$ from Example 3.
11. Use a table to solve $250 + 11h = 481$ from QY2.
12. Use a table on a graphing calculator to solve $3x + 5 = -4$.
13. The graph at the right shows the equation $y = 6.2 - 2.15x$.
    a. Draw dotted lines to show how to find an approximate solution to $3 = 6.2 - 2.15x$.
    b. What is the approximate solution?

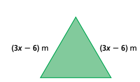


In 14 and 15, use the information provided to write an equation involving the variable. Then use any method to find the value of the variable.

14. The perimeter of the rectangle at the right is 46 feet.



15. The perimeter of the triangle at the right is 97 meters.



REVIEW

In 16–18, graph the equation on a calculator. Does the graph appear to be a line? You may have to adjust the window to view the graph. (Lesson 3-1)

16. $y = 0.402(x - 3) + 1.97x - 0.567$
17. $h = 7 - 2t^2 + t$
18. $y = 4x(5 + x)$

19. Sierra redwood trees are known to be among the world's tallest trees, growing an average of 3.5 feet each year until they mature. If a redwood tree is now 28 feet tall, its height *h* after *y* years is described by $h = 28 + 3.5y$. **(Lesson 3-1)**
    a. Make a table of values for this relationship.
    b. Draw a graph of this situation.
    c. How tall will the tree be in 21 years?
    d. How many years will it take the tree to grow to a height of 259 feet?
20. *Internet World Stats* reported in the summer of 2005 that there were about 220 million Internet users in North America. That number was growing by an average of 23 million users per year. Suppose this rate continues between 2005 and 2010. **(Lessons 3-1, 1-5)**
    a. Make a table showing the total number of millions of Internet users 0, 1, 2, 3, 4, and 5 years after 2005.
    b. Let *x* represent the number of years after 2005. What window on your graphing calculator would be most appropriate to view the graph?
    c. Draw the graph using your graphing calculator.

Many Sierra redwoods are between 250 and 300 feet tall, the tallest being about 325 feet high.

Source: California Department of Parks & Recreation

In 21 and 22, a number is given.
a. Find its opposite.
b. Find its reciprocal. (Lessons 2-8, 2-5)

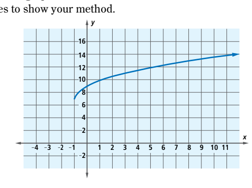
21. 8.3
22. $-\frac{26}{5}$

In 23 and 24, evaluate the expression $\frac{a+b}{3} + 3(a - b)$ for the given values of *a* and *b*. (Lesson 1-1)

23. $a = 11$ and $b = -4$
24. $a = 11x$ and $b = 55x$

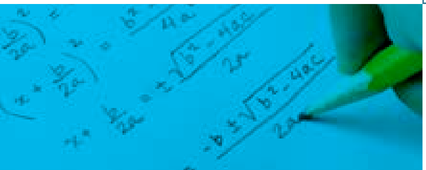
EXPLORATION

25. Use the graph below to solve $11 = 7 + 2\sqrt{x + 1}$. Draw dotted lines to show your method.



QY ANSWERS

1. $m = 5$ is a solution because $|-12 \cdot 5 + 19| = |-60 + 19| = |-41| = 41$.
2. 21 hr



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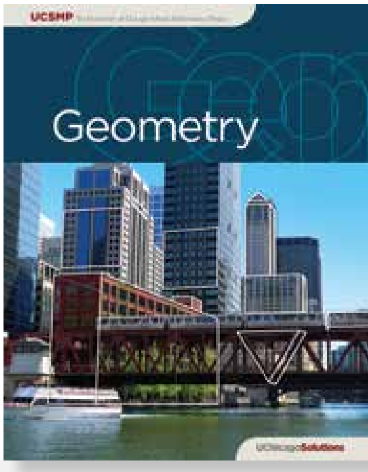
GRADES 8 – 11

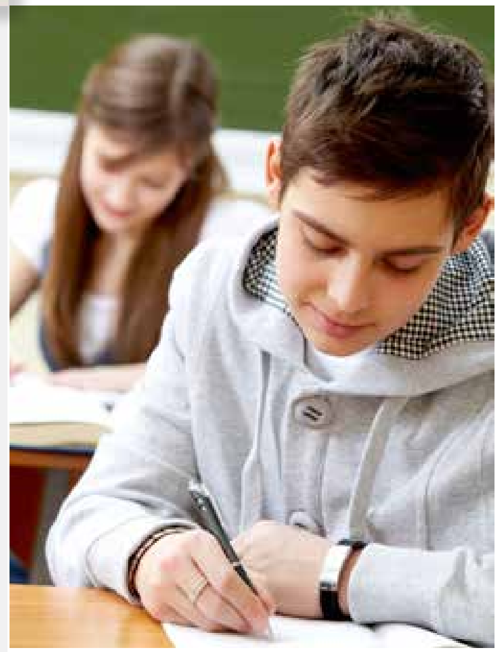
Geometry

SPECIAL LESSONS FOCUS ON GEOMETRY IN ART, ARCHITECTURE, SPORTS, AND MUSIC



The main goal of UCSMP *Geometry* is to provide students with a clear understanding of two-dimensional and three-dimensional figures and the relationships among them.

UCSMP *Geometry* integrates coordinates and transformations throughout, and gives strong attention to measurement formulas and three-dimensional figures. Work with proof writing follows a carefully sequenced development of the logical and conceptual precursors to proof.



UCSMP *Geometry* assumes that students have a graphing calculator and access to a dynamic geometry system (DGS).

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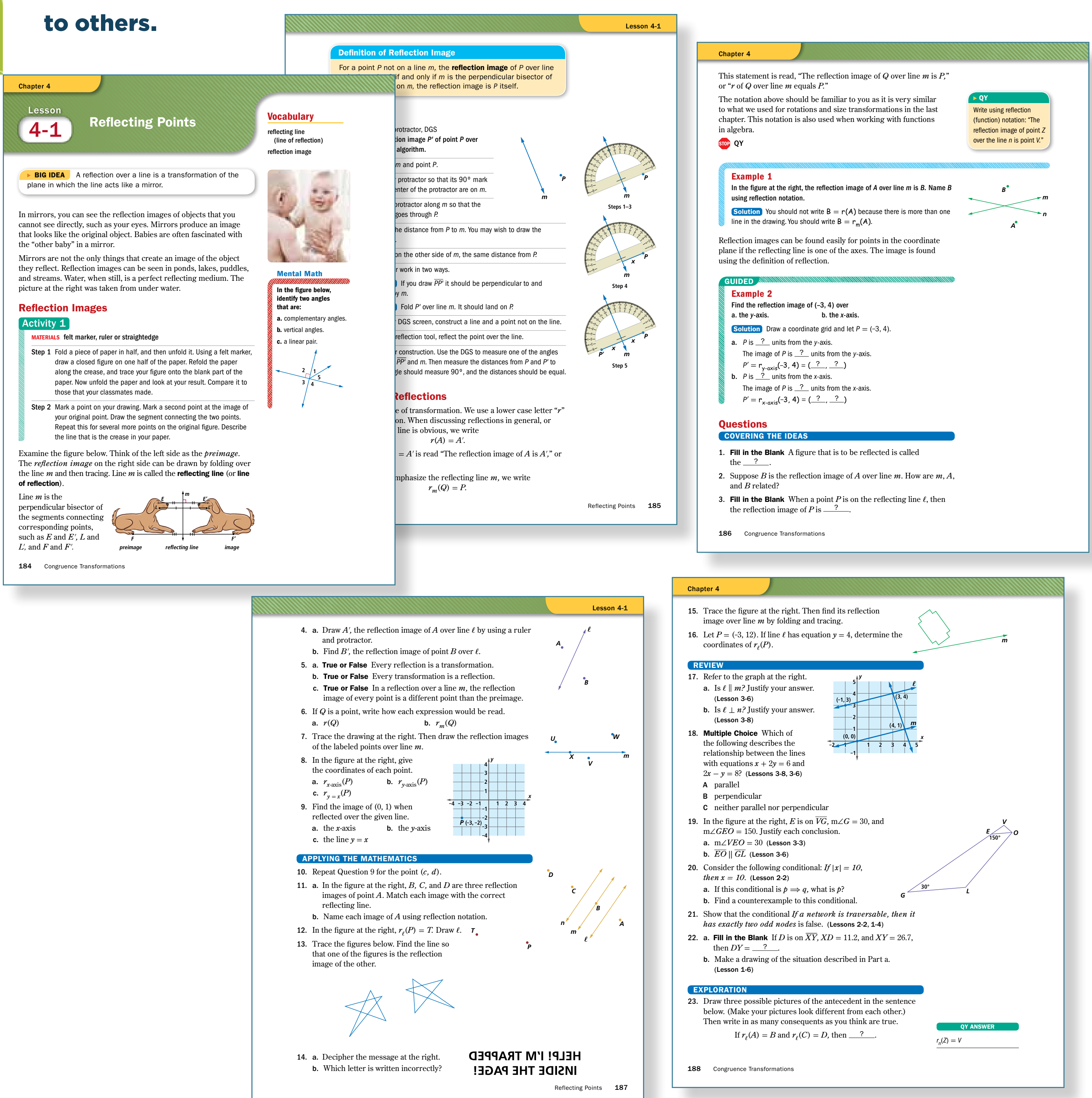
GEOMETRY

Writing helps students clarify their own thinking and is an important aspect of communicating mathematical ideas to others.



Chapter 4

Lesson 4-1 Reflecting Points

BIG IDEA A reflection over a line is a transformation of the plane in which the line acts like a mirror.

In mirrors, you can see the reflection images of objects that you cannot see directly, such as your eyes. Mirrors produce an image that looks like the original object. Babies are often fascinated with the "other baby" in a mirror.

Mirrors are not the only things that create an image of the object they reflect. Reflection images can be seen in ponds, lakes, puddles, and streams. Water, when still, is a perfect reflecting medium. The picture at the right was taken from under water.

Reflection Images

Activity 1

MATERIALS felt marker, ruler or straightedge

Step 1 Fold a piece of paper in half, and then unfold it. Using a felt marker, draw a closed figure on one half of the paper. Refold the paper along the crease, and trace your figure onto the blank part of the paper. Now unfold the paper and look at your result. Compare it to those that your classmates made.

Step 2 Mark a point on your drawing. Mark a second point at the image of your original point. Draw the segment connecting the two points. Repeat this for several more points on the original figure. Describe the line that is the crease in your paper.

Examine the figure below. Think of the left side as the *preimage*. The *reflection image* on the right side can be drawn by folding over the line *m* and then tracing. Line *m* is called the **reflecting line** (or **line of reflection**).

Line *m* is the perpendicular bisector of the segments connecting the corresponding points, such as *E* and *E'*, *L* and *L'*, and *F* and *F'*.

preimage reflecting line image

Vocabulary: reflecting line (line of reflection), reflection image

Mental Math: In the figure below, identify two angles that are: a. complementary angles. b. vertical angles. c. a linear pair.

184 Congruence Transformations

Lesson 4-1

Definition of Reflection Image: For a point *P* not on a line *m*, the **reflection image** of *P* over line *m* is ... if and only if *m* is the perpendicular bisector of ... on *m*, the reflection image is *P* itself.

Reflections

Reflecting Points 185

Chapter 4

This statement is read, "The reflection image of *Q* over line *m* is *P*," or "*r* of *Q* over line *m* equals *P*."

The notation above should be familiar to you as it is very similar to what we used for rotations and size transformations in the last chapter. This notation is also used when working with functions in algebra.

QY: Write using reflection (function) notation: "The reflection image of point *Z* over the line *n* is point *V*."

Example 1: In the figure at the right, the reflection image of *A* over line *m* is *B*. Name *B* using reflection notation.

Solution: You should not write $B = r(A)$ because there is more than one line in the drawing. You should write $B = r_m(A)$.

Reflection images can be found easily for points in the coordinate plane if the reflecting line is one of the axes. The image is found using the definition of reflection.

GUIDED Example 2: Find the reflection image of (–3, 4) over a. the *y*-axis. b. the *x*-axis.

Questions

COVERING THE IDEAS

1. Fill in the Blank A figure that is to be reflected is called the __?__.
2. Suppose *B* is the reflection image of *A* over line *m*. How are *m*, *A*, and *B* related?
3. Fill in the Blank When a point *P* is on the reflecting line ℓ, then the reflection image of *P* is __?__.

186 Congruence Transformations

Lesson 4-1

4. a. Draw *A'*, the reflection image of *A* over line ℓ by using a ruler and protractor. b. Find *B'*, the reflection image of point *B* over ℓ.
5. a. True or False Every reflection is a transformation. b. True or False Every transformation is a reflection. c. True or False In a reflection over a line *m*, the reflection image of every point is a different point than the preimage.
6. If *Q* is a point, write how each expression would be read. a. $r(Q)$ b. $r_m(Q)$
7. Trace the drawing at the right. Then draw the reflection images of the labeled points over line *m*.
8. In the figure at the right, give the coordinates of each point. a. $r_{x\text{-axis}}(P)$ b. $r_{y\text{-axis}}(P)$ c. $r_{y=x}(P)$
9. Find the image of (0, 1) when reflected over the given line. a. the *x*-axis b. the *y*-axis c. the line $y = x$

APPLYING THE MATHEMATICS

10. Repeat Question 9 for the point (*c*, *d*).
11. a. In the figure at the right, *B*, *C*, and *D* are three reflection images of point *A*. Match each image with the correct reflecting line. b. Name each image of *A* using reflection notation.
12. In the figure at the right, $r_\ell(P) = T$. Draw ℓ.
13. Trace the figures below. Find the line so that one of the figures is the reflection image of the other.
14. a. Decipher the message at the right. b. Which letter is written incorrectly?

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Chapter 4

15. Trace the figure at the right. Then find its reflection image over line *m* by folding and tracing.
16. Let *P* = (-3, 12). If line ℓ has equation *y* = 4, determine the coordinates of $r_\ell(P)$.

REVIEW

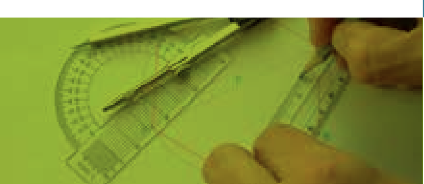
17. Refer to the graph at the right. a. Is ℓ ∥ *m*? Justify your answer. (Lesson 3-6) b. Is ℓ ⊥ *n*? Justify your answer. (Lesson 3-8)
18. Multiple Choice Which of the following describes the relationship between the lines with equations $x + 2y = 6$ and $2x - y = 8$? (Lessons 3-8, 3-6) A parallel B perpendicular C neither parallel nor perpendicular
19. In the figure at the right, *E* is on $\overline{VG}$, m∠*G* = 30, and m∠*GEO* = 150. Justify each conclusion. a. m∠*VEO* = 30 (Lesson 3-3) b. $\overline{EO} \parallel \overline{GL}$ (Lesson 3-6)
20. Consider the following conditional: *If* |*x*| = 10, *then x* = 10. (Lesson 2-2) a. If this conditional is $p \Rightarrow q$, what is *p*? b. Find a counterexample to this conditional.
21. Show that the conditional *If a network is traversable, then it has exactly two odd nodes* is false. (Lessons 2-2, 1-4)
22. a. Fill in the Blank If *D* is on $\overline{XY}$, *XD* = 11.2, and *XY* = 26.7, then *DY* = __?__. b. Make a drawing of the situation described in Part a. (Lesson 1-6)

EXPLORATION

23. Draw three possible pictures of the antecedent in the sentence below. (Make your pictures look different from each other.) Then write in as many consequents as you think are true. If $r_\ell(A) = B$ and $r_\ell(C) = D$, then __?__.

QY ANSWER: $r_n(Z) = V$

188 Congruence Transformations



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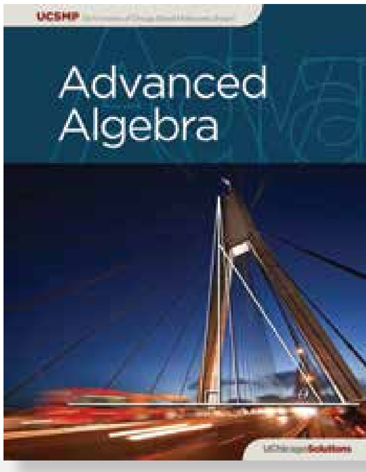
Advanced Algebra

FUNCTIONS PROVIDE A UNIFYING THEME



UCSMP *Advanced Algebra* aims to improve and extend students' existing algebra skills and prepare them for the demands of a second algebra course.

UCSMP *Advanced Algebra* emphasizes facility with algebraic expressions and forms, especially linear and quadratic forms, powers and roots, and functions based on these concepts. Students study logarithmic, trigonometric, polynomial, and other special functions both for their abstract properties and as tools for modeling real-world situations. A geometry course or its equivalent is a prerequisite, as geometric ideas are utilized throughout. Technology for graphing and CAS technology is assumed to be available to students.

Table of Contents

The curriculum seeks to upgrade student achievement. Years of research on the materials confirm improved student performance.

Lesson

3-5 Fitting a Line to Data

Vocabulary

linear regression
line of best fit,
least-squares line,
regression line
deviation

BIG IDEA When data are almost on a line, it is often helpful to approximate the data by a line that minimizes the sum of the squares of the distances from the data points to the corresponding points on the line.

Mental Math

Find the slope of the line.

a. $y = 2.5x + 7$
b. $y = 14$
c. $y - 0.2 = 0.144(x - 0.4)$
d. $16x + 20y = 45$

Square Grabber, which may be provided by your teacher, is a fun game with simple rules. You control a black square. Your job is to capture the other black squares by touching them while avoiding the red squares.

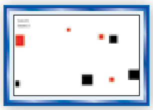
Play the game once. At the end of the game you are told the number of squares you captured and are given a score.

You can write the numbers as an ordered pair (number of squares, score). If you play many times and generate many ordered pairs, you can graph the ordered pairs and use the graph to find a model for the relationship between the number of squares you capture and your score. Is this a simple linear relationship?

Activity 1

MATERIALS internet connection

Step 1 Play Square Grabber 15 times. Record the number of squares you capture and your score for each game in a table like the one below.



Number of Squares (n)	Score (s)
43	855
23	461
12	235

Step 2 Graph your data. Let n = number of squares and s = score. Use n as the independent variable and s as the dependent variable. Does it appear that a line could be a good model for your data?

Step 3 Eyeball a line that comes close to modeling all of the points in your data set. Draw it on your graph.

Step 4 Estimate the coordinates of any two points on your line. (They do not have to be actual data points.) Find an equation for the line through these points.

Step 5 Your equation from Step 4 can be used to estimate your score based on the number of squares you captured. Use your model to estimate your score for capturing 100 squares.

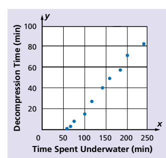
The Regression Line

In Activity 1, you approximated a *line of best fit* for your data by eye. How can you tell which line fits the data the *best*? If you passed your graph of ordered pairs around the room and asked each of your classmates to find a *line of best fit*, you might get as many different lines as you have classmates.

To solve this problem, statisticians have developed a method called **linear regression** that uses all the data points to find the line. A line found by using regression is what people call the **line of best fit**. It is also called the **least-squares line**, or simply the **regression line**. You will learn how regression works in a later course. For now, to find the line of best fit, use a statistics application. For details on how your application works, check the manual or ask your teacher.

Activity 2

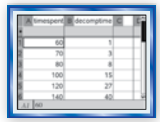
Navy divers who remain underwater for long periods of time cannot come quickly back to the surface due to the high pressure under the water. They must make what are known as *decompression stops* on the way up. If divers skip this procedure they risk a serious medical condition known as *the bends*. The U.S. Navy has created tables that allow divers to know when and for how long they should stop on the way to the surface. The table below gives the decompression time needed (including ascent time) based on how many minutes were spent at a given depth. The points in the table are graphed at the right. Calculate the regression line for the decompression data.



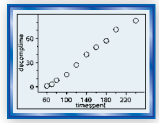
Time Spent at a Maximum Depth of 60 feet (min)	60	70	80	100	120	140	160	180	200	240
Decompression Time Needed (min)	1	3	8	15	27	40	49	57	71	82

(continued on next page)

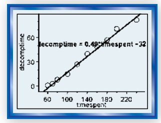
Step 1 Enter the data into columns in your statistics or spreadsheet application. Name the first column *timespent* and the second column *decomptime* to indicate which is the independent and which is the dependent variable.



...of the data. You may be prompted to ...o use for the independent variable ... column to use for the dependent

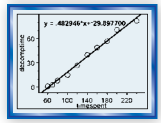


...vable line to your scatterplot. Adjust ...e to eyeball a line of best fit. Record ...movable line. Also record the predicted ...and when $x = 220$.



...e line. Choose to show a linear ...pplication will graph the regression line ...the data. Record the equation of the ...does your movable line compare to the

...le line's predictions to those of the ... $x = 220$, by how much do the ...r from each other? When $x = 80$, by ...redicted value differ from the value in



...that the regression line is a ...a.

...ndent variable, the difference between ...dent variable and the value predicted ...**viation**. The line of best fit has the ...*of the squares of the deviations of* ...*actual values is the least among all* ...*del the data*. This is why it is called the

Questions

COVERING THE IDEAS

In 1–3, refer to Activity 1.

1. A student playing Square Grabber recorded a data point of (48, 985). What does this ordered pair mean in this context?
2. a. Find the regression line for the data you collected in Activity 1.
   b. Pick two values of x to compare how well the regression line predicts y-values compared to the line you eyeballed.
3. A person found $y = 19.0x + 35$ to be an equation for the line of best fit for Activity 1.
   a. What is the slope of the line?
   b. What does the slope mean in this situation?
   c. What is the y-intercept of the line?
   d. What does the y-intercept mean in this situation?
   e. Does the x-intercept have a practical meaning in this case?
4. Refer to Activity 2. If a diver spent 130 minutes at a maximum depth of 60 feet, estimate his decompression time using the regression line.

APPLYING THE MATHEMATICS

5. a. Use regression to find an equation of the line through the points (1, 4) and (-2, 8).
   b. Verify your equation in Part a by finding the slope of the line and using the Point-Slope Theorem.
6. The table below gives the total payroll for the Chicago Cubs from 1998 to 2006.

Year	2006	2005	2004	2003	2002	2001	2000	1999	1998
Payroll (millions of dollars)	94.4	87.0	90.6	79.9	75.7	64.5	62.1	55.4	49.4

Source: http://content.usatoday.com/sports/baseball/salaries/teamresults.aspx?team=17

   a. Draw a scatterplot and eyeball a line of best fit to the data.
   b. Which data point has the greatest deviation from your line?
   c. Find an equation for the regression line for the data.
   d. Which data point has the greatest deviation from the regression line?
   e. Use the regression line and your line to predict the 2007 Chicago Cubs payroll.
   f. Which gives the closer prediction to the actual value of 99.7 million dollars, your eyeballed line or the regression line?

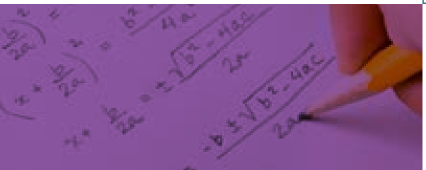
BMI
26.6
27.5
28.4
28.7

REVIEW

10. a. Find an equation for the line that passes through the points (2, 5) and $\left(-\frac{4}{3}, 9\right)$.
    b. Graph the line from Part a. **(Lesson 3-4)**
11. a. Write an equation of a line with slope $-\frac{5}{2}$ and y-intercept 3.
    b. Write an equation of a line parallel to the line in Part a that passes through the point (-1, -2). **(Lessons 3-4, 3-1)**
12. Consider the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$. If the radius of a sphere is divided by three, how many times smaller is the volume of the resulting sphere? **(Lesson 2-3)**
13. Consider the sequence $H_n = (-1)^n(n + 5)$, for integers $n \geq 1$. **(Lesson 1-8)**
    a. Write the first four terms.
    b. Without explicitly calculating, is the 25th term positive or negative?
14. Jayla's cake company charged \$25 per cake, and each week she paid \$50 for supplies and the upkeep of her equipment. She found that $I = 25c - 50$ is an explicit formula for her income I based on the number of cakes c that she sold. **(Lesson 1-5)**
    a. Use a graphing utility to generate a table showing the number of cakes sold and Jayla's income.
    b. Graph the first six data values from your table on coordinate axes.
15. Given that $b: x \rightarrow \frac{3x + 5}{7 - 8x}$ evaluate **(Lesson 1-3)**
    a. $b(2)$.
    b. $b(0)$.
    c. $b(a)$.
    d. $b(-2) + b(-4)$.

EXPLORATION

16. When you calculate a regression equation, some calculators and software include an extra statistic r called *correlation* along with the slope and intercept. Find out what correlation means and what it tells you about the regression line. Go back and examine r for the data sets in Questions 6–8 and see if you can interpret its value in the context of the problems.



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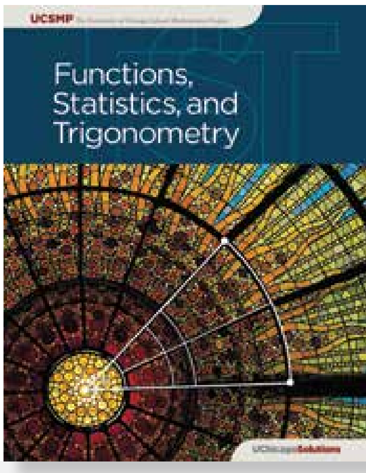
GRADES 10 – 12

Functions, Statistics, and Trigonometry

STATISTICS ARE INTRODUCED IN REAL-WORLD APPLICATIONS



Functions, Statistics, and Trigonometry **presents topics from these three areas in a unified way to help students prepare for everyday life and future courses in mathematics. Spreadsheet, graphing, and CAS technology are employed to enable students to explore and investigate, and to deal with complicated functions and data.**

Functions, Statistics, and Trigonometry (FST) integrates statistics and algebra concepts, and previews calculus in work with functions and intuitive notions of limits. Enough trigonometry is available to constitute a standard precalculus course in the areas of trigonometry and circular functions. Technology is assumed available for student use in graphing, algebraic manipulation, modeling and analyzing data, and simulating experiments.

Table of Contents

The text extends student knowledge of linear, quadratic, exponential, logarithmic, polynomial, and trigonometric equations and functions.

FUNCTIONS, STATISTICS, AND TRIGONOMETRY

Chapter 3

QY

Horizontal Scale Changes

Replacing the variable y by ky in an equation results in a vertical scale change. What happens when the variable x is replaced by $\frac{x}{2}$? By $4x$?

QY

Replacing y by $\frac{y}{4}$ in the equation for $f1$ yields an equation equivalent to $y = \underline{\ ?\ }$. What is the effect on the graph of f?

Activity 2

Lesson 3-5

Graph Scale-Change Theorem

Given a preimage graph described by a sentence in x and y, the following two processes yield the same image graph:
(1) replacing x by $\frac{x}{a}$ and y by $\frac{y}{b}$ in the sentence;
(2) applying the scale change $(x, y) \to (ax, by)$ to the preimage graph.

Proof Name the image point (x', y'). So $x' = ax$ and $y' = by$. Solving for x and y gives $\frac{x'}{a} = x$ and $\frac{y'}{b} = y$. The image of $y = f(x)$ will be $\frac{y}{b} = f\left(\frac{x}{a}\right)$. The image equation is written without the primes.

Unlike translations, scale changes do not produce congruent images unless $a = b = 1$. Notice also that multiplication in the scale change corresponds to division in the equation of the image. This is analogous to the Graph-Translation Theorem in Lesson 3-2, where addition in the translation $(x, y) \to (x + h, y + k)$ corresponds to subtraction in the image equation $y - k = f(x - h)$.

GUIDED

Example 1

The relation described by $x^2 + y^2 = 25$ is graphed at the right.
a. Find images of points labeled A–F on the graph under $S: (x, y) \to (2x, y)$.
b. Copy the circle onto graph paper; then graph the image on the same axes.
c. Write an equation for the image relation.

Solution
a. Copy and complete the table below.
b. Plot the preimage and image points on graph paper and draw a smooth curve connecting the image points. A partial graph is drawn below.
c. According to the Graph Scale-Change Theorem, an equation for an image under $S: (x, y) \to (2x, y)$ can be found by replacing x by $\underline{\ ?\ }$ in the equation for the preimage. The result is the equation $\underline{\ ?\ }$.

	Preimage		Image	
Point	x	y	$2x$	y
A	5	0	10	0
B	?	?	?	?
C	?	?	?	?
D	?	?	?	?
E	?	?	?	?
F	?	?	?	?

The Graph Scale-Change Theorem 181

Lesson 3-5

Lesson **3-5** The Graph Scale-Change Theorem

Vocabulary
horizontal and vertical scale change, scale factor
size change

BIG IDEA The graph of a function can be scaled horizontally, vertically, or in both directions at the same time.

Vertical Scale Changes

Consider the graph of $y = f1(x) = x^3 + 3x^2 - 4x$ shown both in the graph and function table below. What happens when you multiply all the y-values of the graph by 2? What would the resulting graph and table look like? Activity 1 will help you answer these questions.

Mental Math
What is $R_{270°}(1, 0)$?

Activity 1

MATERIALS Graphing utility or slider graph application from your teacher

Step 1 Graph $f1(x) = x^3 + 3x^2 - 4x$ with window $-11 \le x \le 13$ and $-10 \le y \le 40$.

Step 2 Graph $f2(x) = 3(x^3 + 3x^2 - 4x) = 3 \cdot f1(x)$ on the same axes. Fill in the table of values for $f1(x)$ and $f2(x)$ only. Describe how the $f2(x)$ values relate to the $f1(x)$ values.

x	$f1(x)$	$f2(x)$	$0.5 \cdot f1(x)$	$1.5 \cdot f1(x)$	$2 \cdot f1(x)$
-4	?	?	?	?	?
2	?	?	?	?	?
7	?	?	?	?	?
10	?	?	?	?	?

Step 3 Repeat Step 2 with $f3(x) = b(x^3 + 3x^2 - 4x)$ and use a slider to vary the value of b. Set the slider to 0.5, 1.5, and then 2. For each of these b-values, use a function table to fill in a column of the table in Step 2. Describe how the $f3(x)$ values relate to the corresponding values of $f1(x)$.

In Step 2 of Activity 1, we say that the graph of $f2$ is the image of the graph of $f1$ under a *vertical scale change* of magnitude 3. Each point on the graph of $f2$ is the image of a point on the graph of f under the mapping $(x, y) \to (x, 3y)$. You can create the same change by replacing y with $\frac{y}{3}$ in the equation for $f1$, because $\frac{y}{3} = x^3 + 3x^2 - 4x$ is equivalent to $y = 3(x^3 + 3x^2 - 4x)$, or $y = 3f1(x)$ in function notation. Similarly, if you replace y with $2y$ in the original equation, you obtain $2y = x^3 + 3x^2 - 4x$, which is equivalent to $y = \frac{1}{2}(x^3 + 3x^2 - 4x)$ or $y = \frac{1}{2}f(x)$.

The Graph Scale-Change Theorem 179

Chapter 3

Example 2

A graph and table for $y = f(x)$ are given at the right. Draw the graph of $\frac{y}{3} = f(2x)$.

x	$f(x)$
-6	2
-3	-1
0	-1
2	3
6	0

Solution Rewrite $\frac{y}{3} = f(2x)$ as $\frac{y}{3} = f\left(\frac{x}{\frac{1}{2}}\right)$. By the Graph Scale-Change Theorem, replacing x by $\frac{x}{\frac{1}{2}}$ and y by $\frac{y}{3}$ is the same as applying the scale change $(x, y) \to \left(\frac{1}{2}x, 3y\right)$.

So, $A = (-6, 2) \Rightarrow (-3, 6) = A'$
$B = (-3, -1) \Rightarrow (-3/2, -3) = B'$
$C = (0, -1) \Rightarrow (0, -3) = C'$
$D = (2, 3) \Rightarrow (1, 9) = D'$
and $E = (6, 0) \Rightarrow (3, 0) = E'$.

The graph of the image is shown at the right.

Negative Scale Factors

Notice what happens when a scale factor is -1. Consider the horizontal and vertical scale changes H and V with scale factors equal to -1.

$H: (x, y) \to (-x, y)$ and $V: (x, y) \to (x, -y)$

In H, each x-value is replaced by its opposite, which produces a reflection over the y-axis. Similarly, in V, replacing y by $-y$ produces a reflection over the x-axis. More generally, a scale factor of $-k$ combines the effect of a scale factor of k and a reflection over the appropriate axis.

Questions

COVERING THE IDEAS

1. **True or False** Under every scale change, the preimage and image are congruent.
2. Under a scale change with horizontal scale factor a and vertical factor b, the image of (x, y) is $\underline{\ ?\ }$.
3. Refer to the Graph Scale-Change Theorem. Why are the restrictions $a \neq 0$ and $b \neq 0$ necessary?
4. If S maps each point (x, y) to $\left(\frac{x}{2}, 6y\right)$, give an equation for the image of $y = f(x)$ under S.

182 Transformations of Graphs and Data

5. Consider the function $f1$ used in Activities 1 and 2.
a. Write a formula for $f1\left(\frac{x}{3}\right)$.
b. How is the graph of $y = f1\left(\frac{x}{3}\right)$ related to the graph of $y = f1(x)$?
6. **Multiple Choice** Which of these transformations is a size change?
A $(x, y) \to (3x, 3y)$ B $(x, y) \to (3x, y)$
C $(x, y) \to (x + 3, y + 3)$ D $(x, y) \to \left(\frac{x}{3}, y\right)$
7. Functions f and g with $f(x) = -2x^2 + 5x + 3$ and $g(x) = \frac{1}{2}f(x)$ are graphed at the right.
a. What scale change maps the graph of f to the graph of g?
b. The x-intercepts of f are at $x = -\frac{1}{2}$ and $x = 3$. Where are the x-intercepts of g?
c. How do the y-intercepts of f and g compare?
d. The vertex of the graph of f is (1.25, 6.125). What is the vertex of the graph of g?
8. Consider the parabola with equation $y = x^2$. Let $S(x, y) = \left(2x, \frac{y}{2}\right)$.
a. Find the images of (-3, 9), (0, 0), and $\left(\frac{1}{2}, \frac{1}{4}\right)$ under S.
b. Write an equation for the image of the parabola under S.
9. The graph of a function f is shown at the right.
a. Graph the image of f under $S(x, y) = \left(\frac{1}{2}x, 3y\right)$.
b. Find the x- and y-intercepts of the image.
c. Find the coordinates of the point where the y-value of the image of f reaches its maximum.
10. Give another name for the horizontal scale change of magnitude -1.
11. Describe the scale change that maps the graph of $y = \sqrt{x}$ onto the graph of $y = \sqrt{\frac{x}{12}}$.

APPLYING THE MATHEMATICS

12. Refer to the parabolas at the right. The graph of g is the image of the graph of f under what
a. horizontal scale change? b. vertical scale change?
c. size change?
13. Write an equation for the image of the graph of $y = x + \frac{1}{x}$ under each transformation.
a. $S(x, y) = (2x, 2y)$ b. $S(x, y) = \left(\frac{x}{3}, -y\right)$
14. A scale change maps (10, 0) onto (2, 0) and (-5, 8) onto (-1, 2). What is the equation of the image of the graph of $f(x) = x^3 - 8$ under the scale change?

The Graph Scale-Change Theorem 183

Chapter 3

In 15 and 16, give a rule for a scale change that maps the graph of f onto the graph of g.

15. 16.

REVIEW

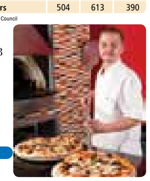
In 17 and 18, an equation for a function is given. Is the function odd, even, or neither? If the function is odd or even, prove it. (Lesson 3-4)

17. $f(x) = (5x + 4)^3$ 18. $g(x) = 5x^4 + 4$
19. If $f(x) = -g(x)$ for all x in the common domain of f and g, how are the graphs of f and g related? (Lesson 3-4)
20. One of the parent functions presented in Lesson 3-1 has a graph that is not symmetric to the x-axis, y-axis, or origin. It has the asymptote $y = 0$. Which is it? (Lesson 3-1)
21. The table at the right shows the number of injuries on different types of rides in amusement parks in the U.S. in 2003, 2004, and 2005. Use the table to explain whether each statement is supported by the data. (Lesson 1-1)
a. Injuries on children's rides decreased slightly each year.
b. The number of injuries on roller coasters decreased from 2003 to 2004.
c. Roller coasters are not as safe as children's rides.

	2003	2004	2005
Total	1954	1648	1713
Children's Rides	277	219	192
Family and Adult Rides	1173	806	1131
Roller Coasters	504	613	390

Source: National Safety Council

22. Pizza π restaurant made 300 pizzas yesterday; 64% of the pizzas had no toppings, 10% of the pizzas had two toppings, and 26% had more than two toppings. How many pizzas had at least two toppings? (Previous Course)

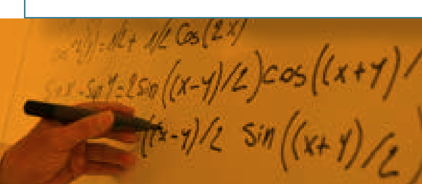


EXPLORATION

23. a. Explore $g(x) = b(x^3 + 3x^2 - 4x)$ for $b < 0$. Explain what happens to the graph of g as b changes.
b. Explore $h(x) = \left(\frac{x}{a}\right)^3 + 3\left(\frac{x}{a}\right)^2 - 4\left(\frac{x}{a}\right)$ for $a < 0$. Explain what happens to the graph of h as a changes.

QY ANSWER
$4(x^3 + 3x^2 - 4x)$; a vertical scale change of magnitude 4

184 Transformations of Graphs and Data



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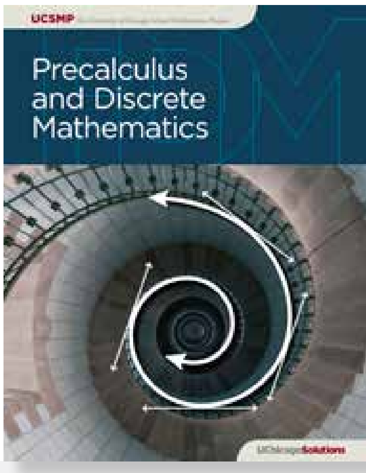
GRADES 11 – 12

ESTABLISHES FORMAL MATHEMATICAL REASONING NEEDED FOR COLLEGE-LEVEL COURSES

Precalculus and Discrete Mathematics



Precalculus and Discrete Mathematics integrates the major ideas of mathematics needed for the future study of calculus and presents the fundamental notions of discrete mathematics.

Precalculus and Discrete Mathematics (PDM) integrates the background students must have to be successful in calculus with the discrete mathematics helpful in computer science. It balances advanced work on functions and trigonometry, an introduction to limits, and other calculus ideas with work on number systems, combinatorics, recursion, and graph theory.

Mathematical thinking, including specific attention to formal logic and proof, is a theme throughout. Technology is assumed available for student use in graphing and algebraic manipulation.

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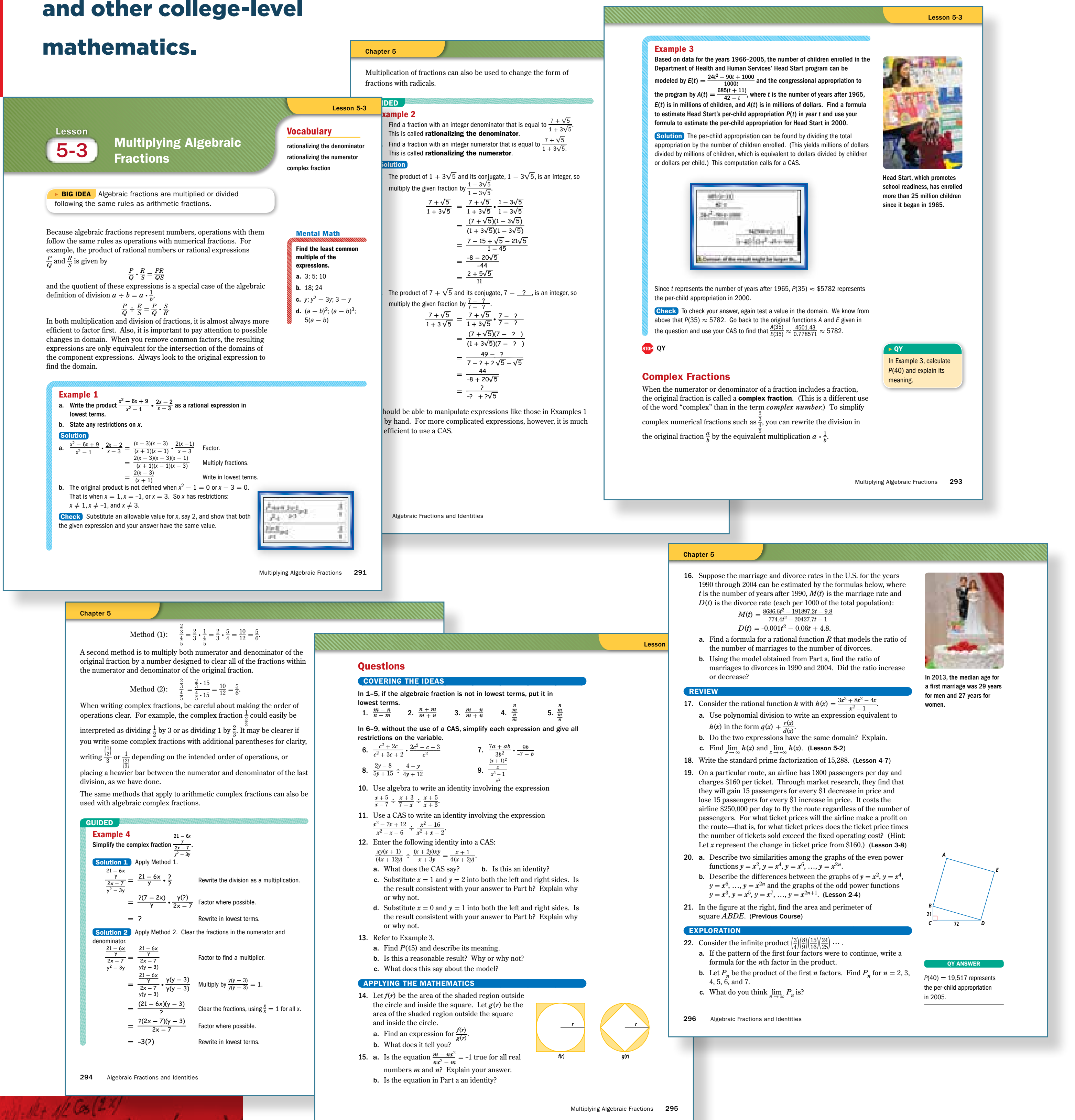

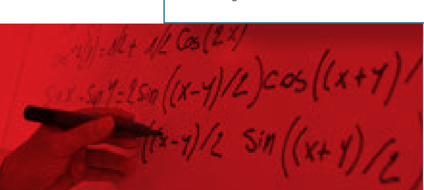

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Payment Terms

All items are invoiced FOB Shipping Point. Payment terms are subject to credit analysis and review.

For sales to schools and other institutions meeting our credit criteria, payment is due Net Invoice.

Orders from individuals or for fewer than 5 items must be prepaid prior to shipping. We accept checks, money orders, wire transfers, and most major credit cards.

Shipping

Orders are transported and delivered by third-party carriers. The University of Chicago is not responsible for their performance.

If you require shipping through a preferred company, check the box on the order form under "Shipping Options" and provide the requested information.

Actual shipping costs are added to your invoice. If you require a firm price for shipping before ordering in order to obtain purchasing authorization, please contact us. Alternatively you may add 22% to the cost of the items you are purchasing and we will bill you the lower of actual costs or the 22%.

Damaged, Defective, or Missing Items

Please examine your shipment as soon as possible after it arrives. You must report any damaged or defective products or missing items within 30 days of receipt.

We will replace defective or missing items at our expense so long as stock is available. If an item is out of stock we will issue a refund.

If an item is damaged we will work with you and the shipping carrier to resolve the matter appropriately.

Other Returns

We accept returns within 90 days of product receipt without prior written authorization. Returns after 90 days are subject to prior authorization. Credit for returned items is subject to the following conditions:

- The items are unused and unmarked and in resalable condition.
- Our invoice, the packing slip, and your order form or purchase order must be included with any returned items.

Return shipping to our distribution center is at your expense. We recommend a traceable form of shipment, as credit will not be given for items lost in transit.

For Order Processing, Customer Service & Returns:

Chicago Distribution Center
11030 S. Langley Avenue
Chicago, IL 60628

Toll-Free: 800.621.2736
Main No.: 773.702.7000
custserv@press.uchicago.edu

UCSMP Order Form

To Place an Order

Toll-Free: 800.621.2736

Scan & Email:
custserv@press.uchicago.edu

BILL TO

Organization Name

Authorized Person

Street Address

Floor/Suite/Office

City ST Zip

Telephone

Email Address

SHIP TO *(only if different)*

Facility Name

Contact Person

Street Address

Floor/Suite/Office

City ST Zip

Telephone

Email Address

PURCHASE AUTHORIZATION

☐ Check here if a Purchase Order or other authorization form is attached.

SHIPPING OPTIONS

☐ Check here if you have specific shipping terms and needs, and provide all information needed to authorize and process your preference on a separate sheet. All other orders are shipped by third-party carriers we select and the cost is added to your bill.

You may omit this page if you are providing us with a purchase order or other official and binding authorization.

13-Digit ISBN	Title	Quantity	Price
	TOTAL		

To Place an Order

Toll-Free: 800.621.2736

Scan & Email: custserv@press.uchicago.edu

UChicagoSolutions

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UCSMP
THE UNIVERSITY OF CHICAGO SCHOOL MATHEMATICS PROJECT

http://ucsmp.uchicago.edu/secondary/overview